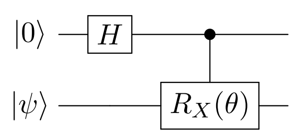
Problem set 1 for 41076: Methods in Quantum Computing

due September 18
15 points

1 Quantum channels (5 points)

Consider the following quantum circuit



where R_x is a rotation along the X-axis.

$$R_X(\theta) = \exp(-iX\theta/2) = \begin{pmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \tag{1}$$

and $\psi = a|0\rangle + b|1\rangle$ is a quantum state on a single qubit.

1. What is the output of this circuit?

2. Now, trace out the first qubit. What is the reduced density matrix corresponding to the second register after the application of the circuit?

3. Looking the at the second register only, write the operation corresponding to the circuit as a quantum channel. What are its Kraus operators?

4. What is the fidelity between $|\psi\rangle\langle\psi|$ and the state on the second qubit?

2 SIC POVM (5 points)

Consider the following quantum states:

$$|v_1\rangle = |0\rangle \tag{2}$$

$$|v_2\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle \tag{3}$$

$$|v_3\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}e^{i\frac{2\pi}{3}}|1\rangle \tag{4}$$

$$|v_4\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}e^{i\frac{4\pi}{3}}|1\rangle \tag{5}$$

$$\tag{6}$$

1. Define $M = \{M_1, M_2, M_3, M_4\}$ where $M_i = \frac{1}{2}|v_i\rangle\langle v_i|$. Show that M is a POVM.

2. Consider a general quantum state ρ parametrized as

$$\rho = \mathbb{1} + r_1 X + r_2 Y + r_3 Z \tag{7}$$

   where X, Y, Z are Paulis. Show how the parameters r_i of ρ will be dependent on the expectation values of elements of M. How can we reconstruct ρ from the measurement statistics of M?

3 Spin in a magnetic field (5 points)

Here we will demonstrate a very simplified way of applying single qubit rotations by applying a magnetic field.

Define a hermitian matrix

$$H = -\frac{\mu B}{2m}\begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} \tag{8}$$

1. Find the eigenvalues and corresponding eigenvectors of H? Which one corresponds to the ground state.

2. Staring in a state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, find the evolution under H.