Methods in quantum computing

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August 26, 2022

University of Technology Sydney

Today

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Methods in quantum computing

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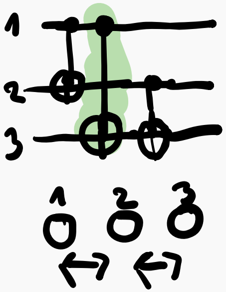
Today

1. Quantum computing stack
2. DiVincenzo's criteria
3. Decoherence in a quantum system
4. Tomography
5. Selected physical architectures - ion traps

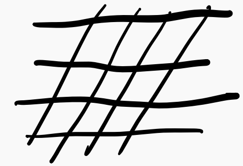
The quantum computing stack



Barenco et al 90s

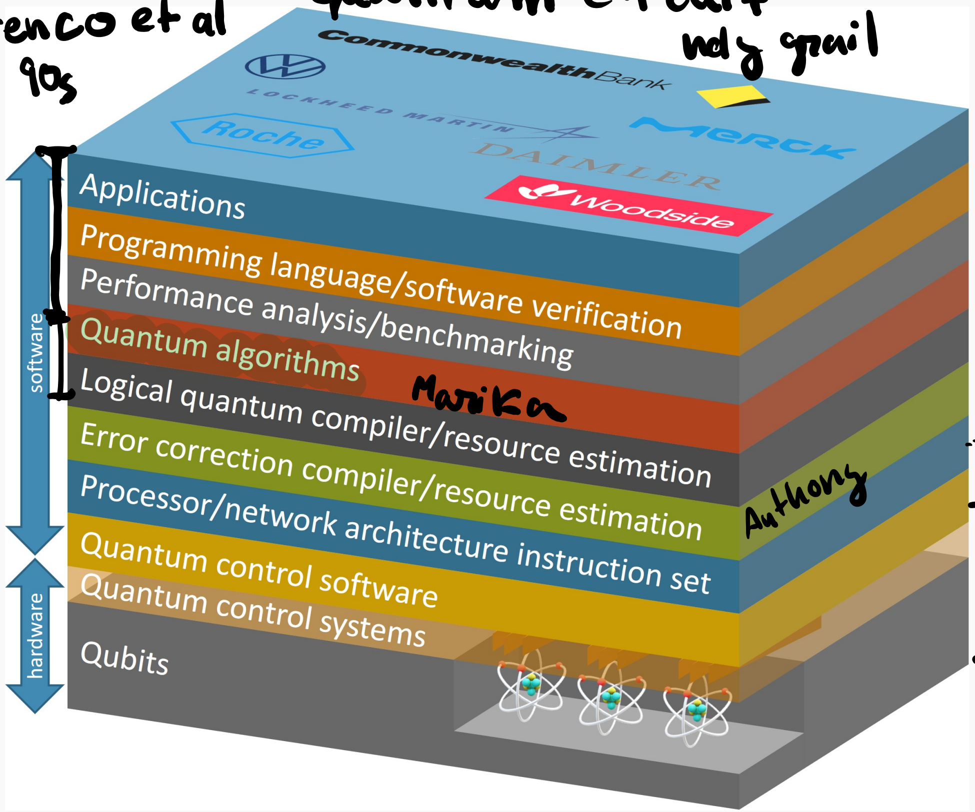
- quantum circuit

holy grail



planar graphs usually

middleware



Marika

Anthony

Lyra

+supporting tech: fridges, control electronics, sources detectors ...

Figure 1: Quantum computing stack. Credit: Michael Bremner

+ supporting technologies

Hardware level

single qubit gates
entangling operations

} set of gates that allows creating any operation
$\{H, T, CNOT\}$

measurement
q. inf → classical information

What do we need to build a quantum computers?

2-state system
$|0\rangle, |1\rangle$

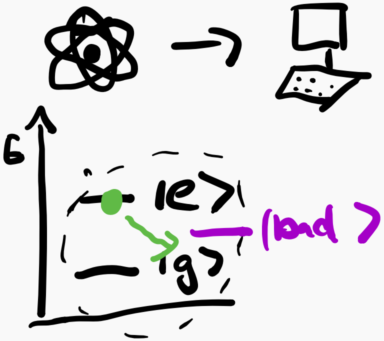
DiVincenzo's criteria

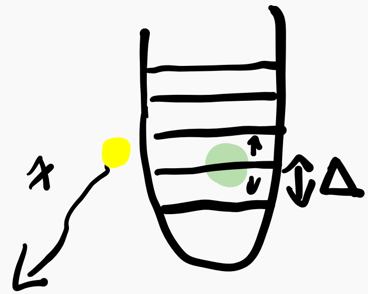
David

1. A scalable physical system with well-characterized qubit
2. The ability to initialize the state of the qubits to a simple fiducial state
3. Long relevant decoherence times
4. A "universal" set of quantum gates
5. A qubit-specific measurement capability

Note that all the requirements need to be satisfied simultaneously.

A scalable physical system with well-characterized qubit





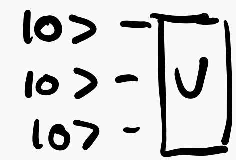
- an isolated 2-dimensional quantum state within a larger system
- always be able to increase the number of qubits in our computer
- practical constraints: physical size of a chip

instead of

2 qubit quantum computer
- electron spin & nuclear spin

3-qubit quantum computer?

The ability to initialize the state of the qubits to a simple fiducial state

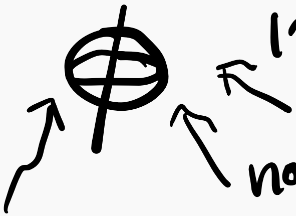


NMR - difficult

How can prepare $|0 \ldots 0\rangle$?

- The second requirement is to be able to initialize the initial state of a quantum computer. A typical state for use in many calculations is the state $|0 \ldots 0\rangle$, which is a pure state. However, contact with the environment leads to decoherence, i.e. noise. A difficulty in some systems is initializing all the qubits very close to the state $|0 \ldots 0\rangle$ without restoring to a measurement. In other systems, the sources that create states are probabilistic, and creating a state with multiple qubits is in practice difficult (for example photonics).

Long relevant decoherence times



$|\psi\rangle \rightarrow 10\,s \quad |\psi\rangle$

noise

$|\psi\rangle - \boxed{U} - \boxed{U} - \boxed{U} -$ ideal

$|\psi\rangle \rightarrow \underset{N}{\boxed{}} - \boxed{U} - \underset{N}{\boxed{}} - \boxed{U} - \underset{N}{\boxed{}} -$

$\rightarrow$ long wrt gate times

A "universal" set of quantum gates

$\{H, T, CNOT\}$

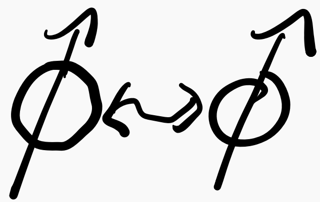
$\{H, Toff\}$

$\{H, T, C\text{-}Z\}$

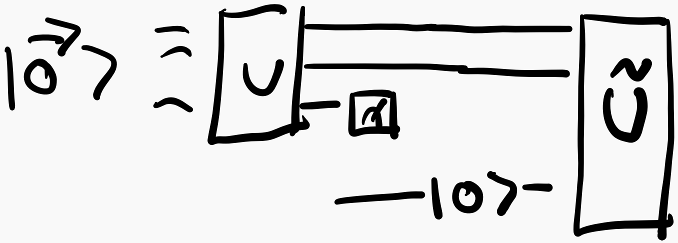
$\{H, T, \sqrt{SWAP}\}$

$\cancel{\{H, T, SWAP\}}$

Rotations along two different axis



A qubit-specific measurement capability

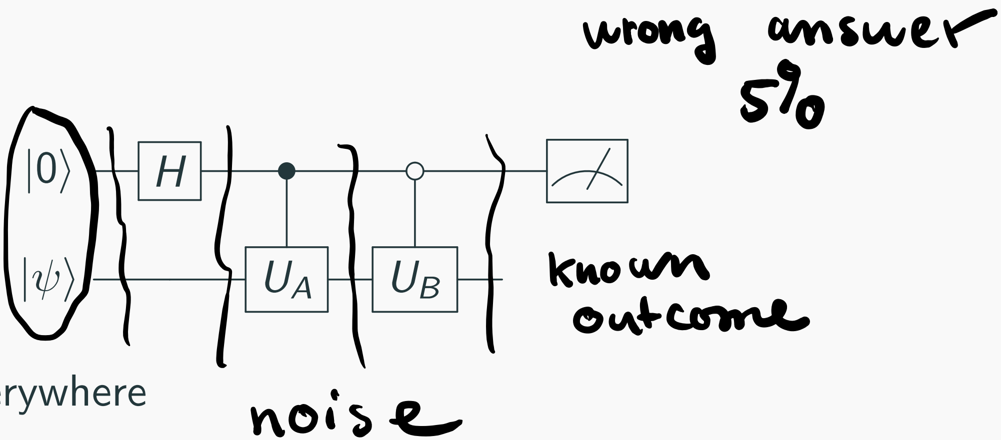


depends on the outcome of the measurement

measure each qubit individually

Quantum hardware

Ideal:



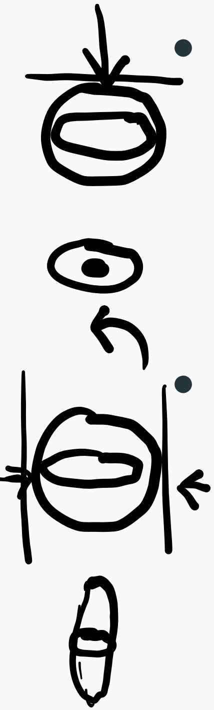
Real life: noise everywhere

Decoherence

How long can $|\psi\rangle$ live inside of a system.

Quantum states decay exponentially. Under the presence of noise, we identify two times, T1 and T2 that give typical time scales for stability of qubits $t \gg T_1$ $|1\rangle$ is not stable

$$|1\rangle \rightarrow p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|$$



- T1 measures how fast a qubit loses energy. Often, the state $|0\rangle$ is encoded to the ground state and $|1\rangle$ into an excited state (i.e. state with higher energy). T1 measures the exponential decay time for a qubit to relax from $|1\rangle$ to $|0\rangle$.

dephasing

- T2 measures the stability of a phase of a qubit. Starting from a particular state on the "equator" of a Bloch sphere, for a time $t \geq T2$ the phase disappears and the mixed state will be along the z (vertical) axis.

$|+\rangle \leftrightarrow |-\rangle$ become indistiguishable

$|0\rangle, |1\rangle$ will stay the same

Noise channels

Dephasing and depolarizing are two simple models. In practice, characterizing what is actually happening is much more difficult.

Quantum tomography

$\mathcal{N}(|0\rangle\langle 0|) \to \rho$?

What state did I obtain from after some unknown channel?

Tomography is the process of learning a (complete) description of a quantum state out of many copies and many measurements.

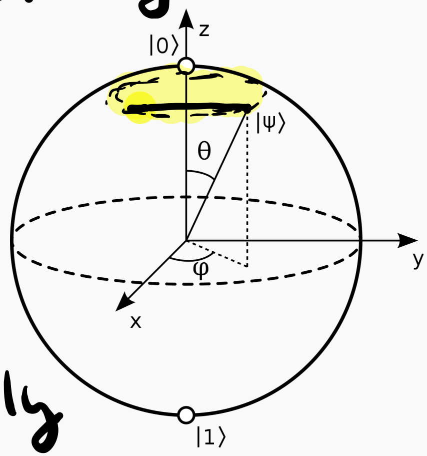
Tomographically complete measurement

To learn a fully quantum-mechanical description of a quantum state, one needs to measure many copies (i.e. thousands for a single qubit) of a quantum state in different basis sets. If a particular measurement allows us to fully reconstruct a quantum state, we say that the measurement is tomographically complete.

→ grows exponentially with the number of qubits

$|\psi\rangle$?

$\{|0\rangle\langle 0|,\ |1\rangle\langle 1|\}$



choose 1, 2, 3 randomly

$\downarrow$ x $\quad \downarrow$ y $\quad \downarrow$ z

Exercise

A single qubit is fully characterized by a vector $\vec{r}, |r| \leq 1$ such that

$$\rho = \frac{1}{2}I + r_0\sigma_x + r_1\sigma_y + r_2\sigma_z. \tag{1}$$

$$= \frac{1}{2}\mathbb{1} + \vec{r}\cdot\vec{\sigma}$$

Take a set of operators

$$M = \left\{\underbrace{\frac{I+X}{6}}_{M_1}, \underbrace{\frac{I-X}{6}}_{M_2}, \frac{I+Y}{6}, \frac{I-Y}{6}, \frac{I+Z}{6}, \frac{I-Z}{6}\right\}. \tag{2}$$

Show that

1. M is a POVM (operators are positive semidefinite and sum to identity). You don't have to work out positivity for all the elements of M.
2. M is tomographically complete, i.e. measuring enough times will allow us to learn the vector $\vec{r}$.

$$\mathrm{Tr}(M_1 \rho) = \mathrm{Tr}\left[\left(\frac{1+X}{6}\right)\left(\frac{\mathbb{1}}{2} + \vec{r}\cdot\vec{\sigma}\right)\right]$$

$$= \frac{1}{6}\mathrm{Tr}\left[\frac{\mathbb{1}}{2} + \vec{r}\cdot\vec{\sigma} + \frac{X}{2} + r_0 \underbrace{X\cdot X}_{\mathbb{1}} + r_1 XY + r_2 XZ\right]$$

$\mathrm{Tr}[X] = \mathrm{Tr}[Y] = \mathrm{Tr}[Z] =$

$$= \quad 1 \qquad 0 \qquad 0 \qquad 2\cdot r_0 \qquad 0 \qquad 0$$

$$= \frac{1}{6} + \frac{r_0}{3} = P_1$$

$$\mathrm{Tr}(M_2 \rho) = \frac{1}{6} - \frac{r_0}{3} = P_2$$

$\rightarrow P_1, P_2, P_3, P_4, P_5, P_6$ I know how often I get each outcome.

$$P_1 - P_2 = \frac{2}{3} r_0$$

$$r_0 = \frac{3}{2}(P_1 - P_2)$$

$$r_1 = 3/2\,(P_3 - P_4)$$

$$r_2 = 3/2\,(P_5 - P_6)$$

Noisy gates

Process tomography helps us to learn what operations we actually performed.

Want to apply Z gate.

$$\text{"}Z\text{"}|0\rangle = |0\rangle$$

$$\text{"}Z\text{"}|1\rangle = |1\rangle \checkmark$$

You were applying $\mathbb{1}$ the whole time.

input

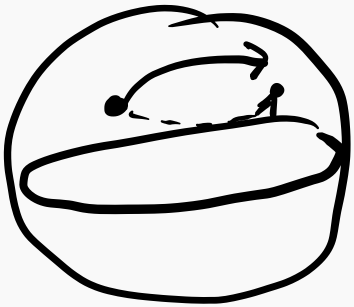
State dependent. $(U - \tilde{U})|\psi\rangle$

depends on the state

$$\text{"}Z\text{"}|0\rangle = |\psi\rangle$$

seems like a perfect Z gate

Gate fidelity



Fidelity - how close we are from a desired state

(Average) gate fidelity - how far our gate took us from a desired state, average over all possible initial states.

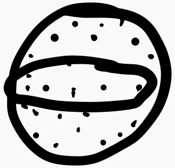
Typical fidelities

In 2022, 99% fidelity for 2-qubit gates and 99.9% for single qubits gates are considered to be very good numbers.

Exercise: Suppose you have a 99% of percent of success when performing an operation. How many operations in sequence can you perform before the chance of successfully performing the sequence gets below 50%? You can assume that the errors are independent.

$(0.99)^n$

Gate fidelity



The average fidelity of a channel is defined with respect to the identity channel

$\langle V\psi | \mathcal{E} | V\psi \rangle$

$$F(\mathcal{E}) = \int d\psi \, \langle\psi| \, \mathcal{E}(\psi) \, |\psi\rangle \quad (3)$$

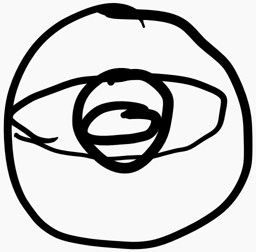
Fidelity between ideal outcome $V|\psi\rangle$ actual outcome $\mathcal{E}(\psi)$

as an average over all state fidelities. To obtain the average, we must integrate over all the quantum states in a given Hilbert space with equal weightings and satisfy $\int d\psi = 1$.

randomized benchmarking

t-desings - prepare a set of states uniformly enoug distributed around the Bloch sphere

Exercise



$\mathcal{E}(|\psi\rangle\langle\psi|)$

Compute the fidelity of a qubit depolarizing channel

$$\mathcal{E}(\rho) = (1-p)\underbrace{|\psi\rangle\langle\psi|}_{\rho} + p\frac{I}{d}.$$

d=2 for a single qubit

$? = \int d\psi \, \langle\psi|\mathcal{E}(\psi)|\psi\rangle$

$\tilde{U}$ faulty unitary U

$$\boxed{\tilde{U}} = \boxed{U}\,\boxed{\text{noise}}$$

fidelity between noise and $\mathbb{1}$

Gate fidelity forunitaries

for unitaries

INF

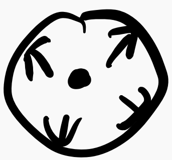
Fid. measures

$\mathbb{1}$ INF actual

$\mathbb{1}$ actual gate Toff

Computing average gate fidelities can be further simplified using Nielsen's formula [?]. In a special case when the channel is unitary, we can compute its fidelity (with respect to the identity channel) as

$$F(U) = \frac{d + |Tr(U)|^2}{d + d^2}. \tag{4}$$



? want to be

Fid. definition

↳ distance between ($\mathbb{1}$, actual gate)

want: (actual gate, Toff.)

$\Downarrow$

$\mathbb{1}$

Exercise

$$\frac{8+6^2}{8+8^2} = 61.1\%$$

V, insted $\tilde{V}$

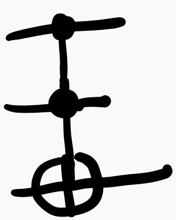
$\tilde{V} = V \circ N$

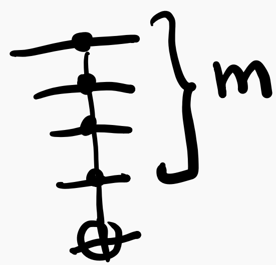
ideal $N = \mathbb{1}$

$N = \text{Toff}$

$F(N, \mathbb{1}) = F(N)$

1. Verify that $F(\mathbb{1})=1$ in (4).
   $\to$ d-dim
2. The hottest quantum startup promises to do quantum computing by implementing Hadamard and Toffoli gates. However, they have a minor issue: their Toffoli gates are not working and they are simply doing nothing (i.e. identity gates). What is the fidelity of their "Toffoli" gate?
   3-qubits, 8 dim
3. What if they replace all m-controlled-NOT gates with the identity?





$$\frac{2^{10}+(2^{10}-2)^2}{2^{10}+(2^{10})^2} \stackrel{?}{=} 99.6\%$$

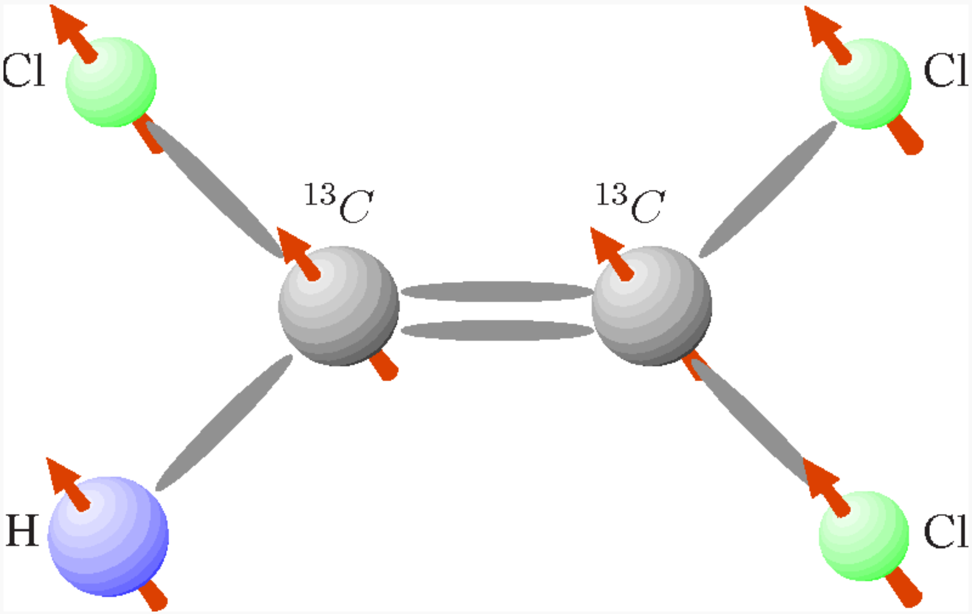
Physical architectures

qubits - physical particles or artificial (advanatges for both)

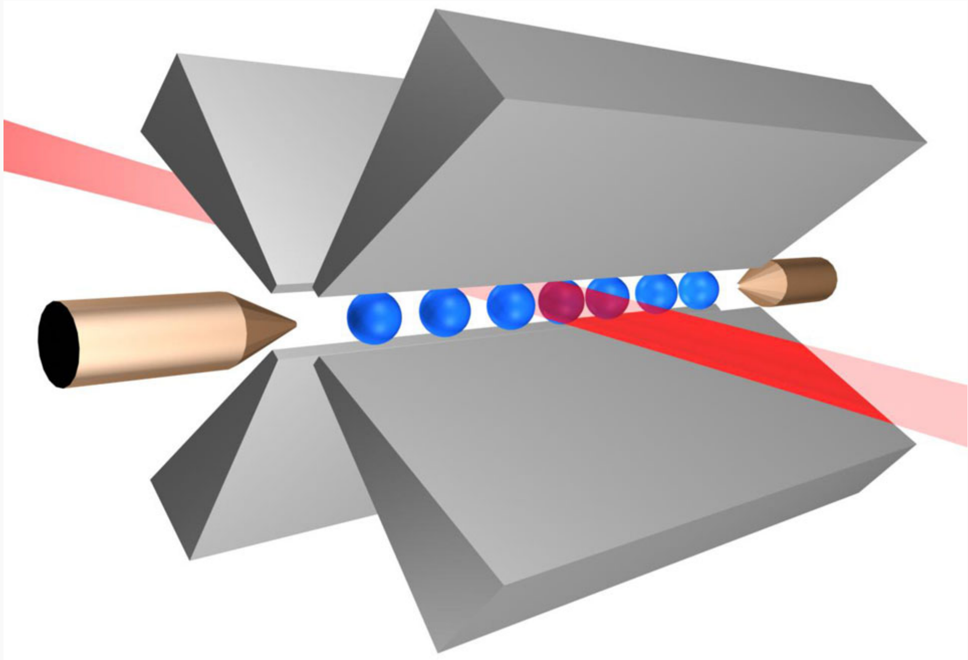
must be a way to satisfy DiVincenzo's criteria

Hardware in 2022

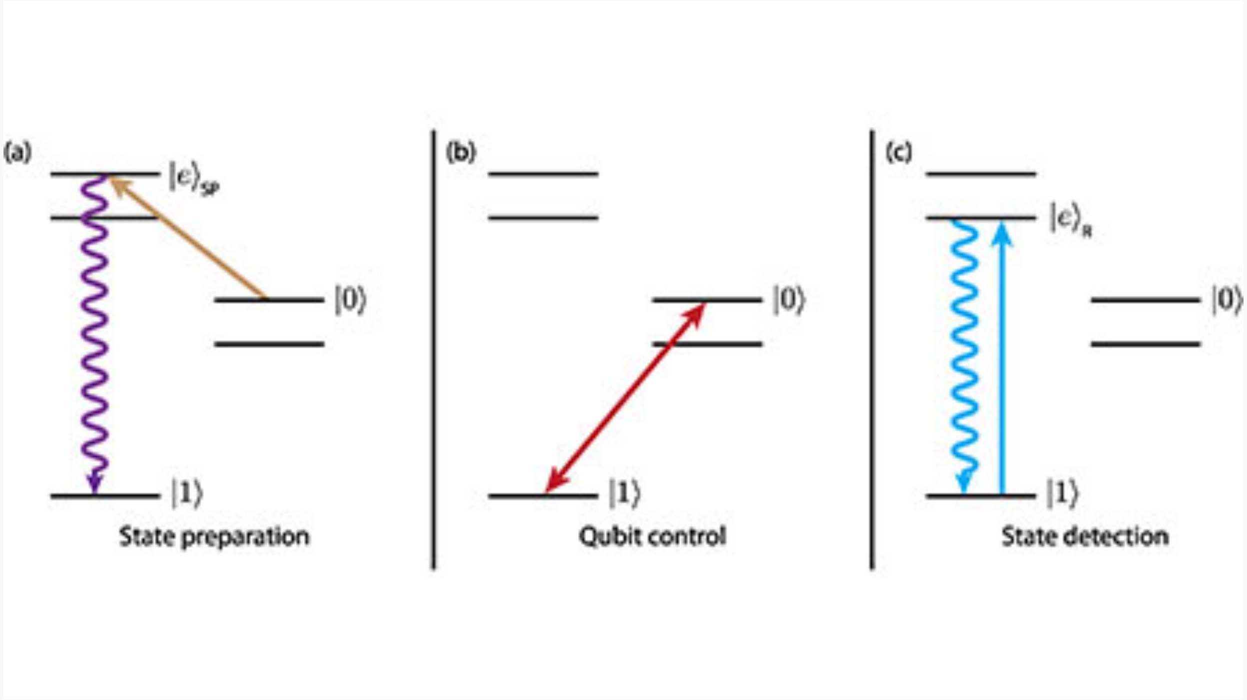
NMR



Ion traps



Ion trap qubits



Strengths and limitations of ion traps

- \+ ions create identical qubits (but the control is not uniform)
- \+ lowest gate errors out of all approaches very slow gates About 50 ions is the maximum for a trap without using individual control. Coupling different traps has not been very successful so far.